CHAPTER 14

PRACTICE EXERCISES

SECTION I MULTIPLE CHOICE

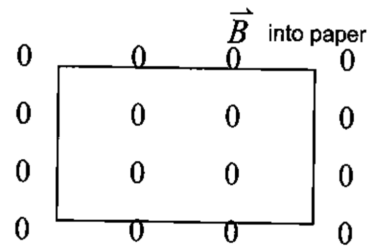

Figure 6

1. A rectangular loop of dimensions 0.04 m by 0.06 m is at rest in a uniform magnetic field of magnitude 0.5 T. The field is perpendicular to the plane of the loop coming out of the page. The magnetic flux through the loop is

(A) $12\ \text{T} \cdot \text{m}^2$ (B) 0 (C) $0.12\ \text{T} \cdot \text{m}^2$ (D) $12 \times 10^{-4}\ \text{T} \cdot \text{m}^2$ (E) $0.5\ \text{T} \cdot \text{m}^2$

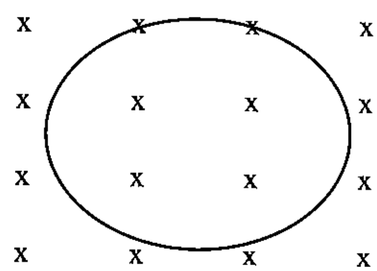

Figure 7

2. A flexible conducting loop is placed in a magnetic field with the plane of the loop perpendicular to the field. Which of the following will NOT induce a current in the loop?

(A) increase the magnitude of the field
(B) stretch the loop, making a larger circle
(C) move the loop parallel to the field
(D) remove the loop from the field
(E) rotate the loop about a diameter

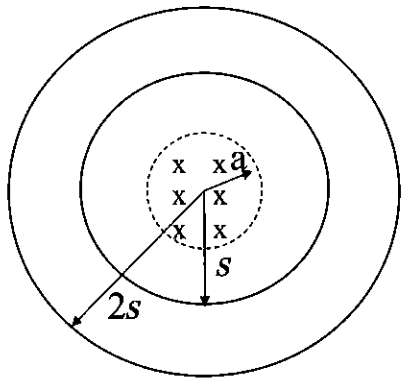

Figure 8

3. Two concentric loops have radii s and $2s$. A spatially uniform magnetic field over the range $r < a$ with $a < s$ is changing at a constant rate. The voltage induced in the outer loop is V_{in}. The voltage induced in the inner loop is

(A) 0 (B) $\frac{V_{in}}{2}$ (C) $\frac{V_{in}}{4}$ (D) $2V_{in}$ (E) V_{in}

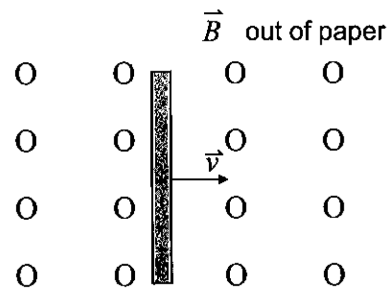

Figure 9

4. A wire of constant length is moving through a uniform magnetic field at a constant velocity with the velocity vector perpendicular to the field. A graph of the induced voltage between the ends of the wire as a function of time would look like

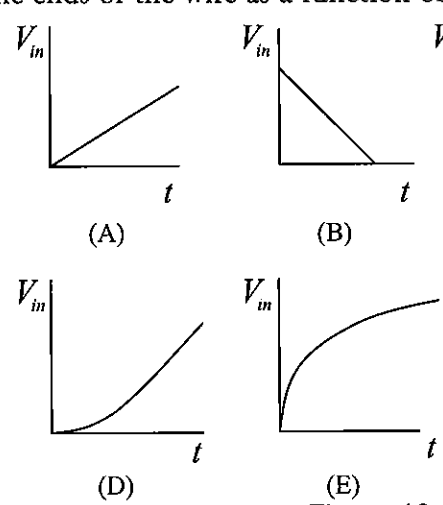

Figure 10

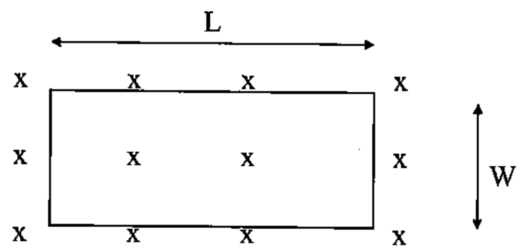

Figure 11

5. A rectangular loop of dimensions L and W and resistance R is stationary in a changing magnetic field. To produce a current I in the loop, the field must change at a rate of

(A) $\frac{LW}{IR}$ (B) $\frac{IR}{LW}$ (C) $\frac{L}{IR}$ (D) $\frac{IRL}{W}$ (E) $\frac{IRW}{L}$

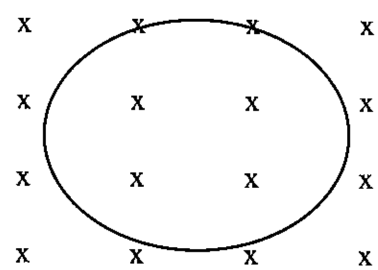

Figure 12

6. A magnetic field, perpendicular into the page to a circular loop, is decreasing as time goes on. In which direction will the current induced in the loop flow?

(A) counterclockwise
(B) clockwise
(C) No current will flow.
(D) out of the page
(E) into the page

Questions 7 and 8

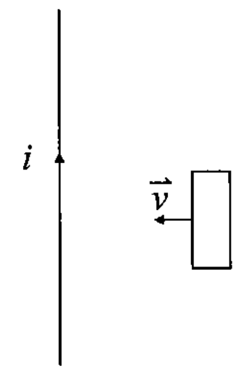

Figure 13

7. A rectangular loop is moving toward a long wire carrying current up as shown in the figure. In which direction will the induced current in the loop flow?

(A) counterclockwise (B) clockwise (C) No current will flow. (D) out of the page (E) into the page

8. The force exerted on the loop by the long wire will be directed

(A) into the page (B) out of the page (C) left (D) right (E) down

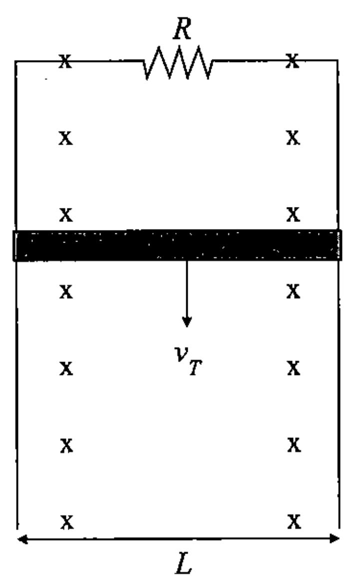

Figure 14

9. A conducting rail of mass m and length L slides vertically along frictionless rails connected by a resistance R. A uniform magnetic field $\vec{B}$ is directed perpendicular to the plane of the rails and into the paper as shown. When the falling rail has reached terminal speed v_T, the rate at which thermal energy appears in the resistor is

(A) mgv_T (B) $\frac{BLv_T}{R}$ (C) $\frac{B^2L^2v_T}{R}$ (D) BLv_T (E) $\frac{mg}{BL}$

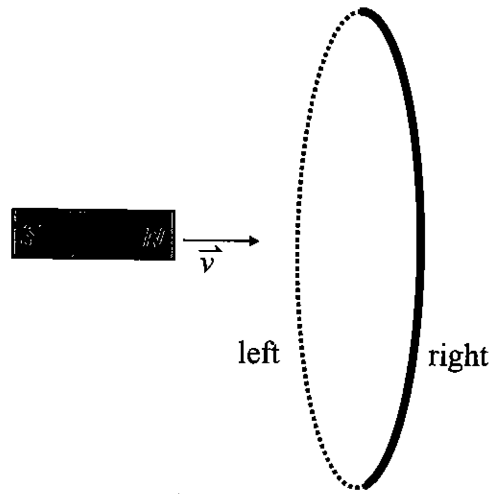

Figure 15

10. A bar magnet moves at constant speed through a circular conducting loop. Viewed from the left, the current flow in the loop will be

(A) clockwise only
(B) counterclockwise only
(C) clockwise first, then counterclockwise
(D) counterclockwise first, then clockwise
(E) Current does not flow in the loop.

CHAPTER 14

PRACTICE EXERCISES

SECTION II FREE RESPONSE

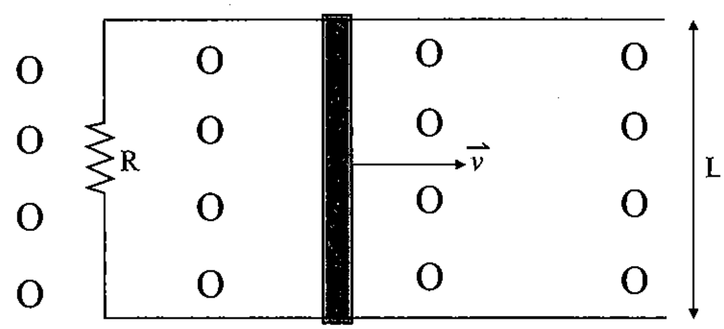

Figure 16

1. A conducting rail, positioned to slide without friction over fixed conducting rails separated by a distance L, is pulled along at a constant speed v by some external force $\vec{F}$. A uniform magnetic field $\vec{B}$ points out of the paper, perpendicular to the plane of the loop. The fixed rails are connected by a resistance R.

   (a) On the diagram, indicate the direction of the induced current flow.
   (b) Determine the magnitude of the induced current.
   (c) Determine the magnitude of force F needed to keep the rail moving at a constant speed.
   (d) The force $\vec{F}$ is suddenly removed. How much energy will be dissipated in the resistor as the rail slows to a stop?

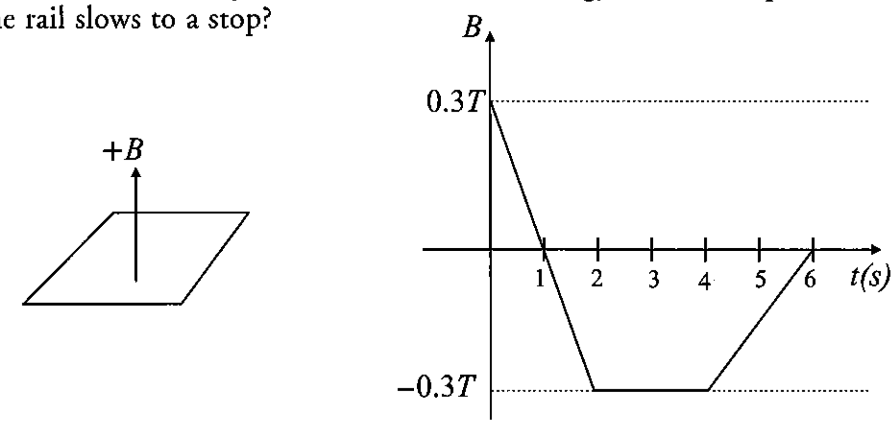

Figure 17

2. A square loop of resistance 0.5 Ω and side 0.6 m is oriented so that the plane of the loop is perpendicular to a magnetic field that is uniform spatially but changing in time as depicted in the graph below. The positive direction of $\vec{B}$ is up.

   (a) Calculate the flux through the loop at $t = 4$ s.
   (b) Find the induced voltage at $t = 5$ s.
   (c) On the axes below, sketch the induced current as a function of time. Indicate numerical values on the current axis and assume counterclockwise current is positive.

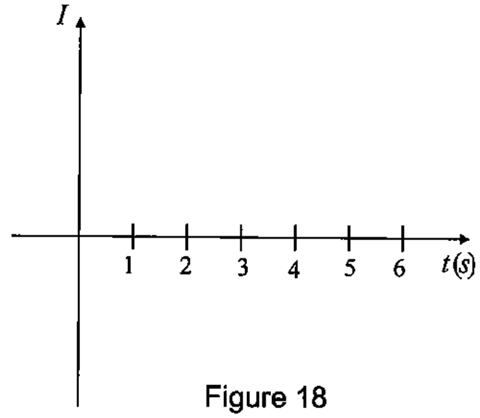

Figure 18

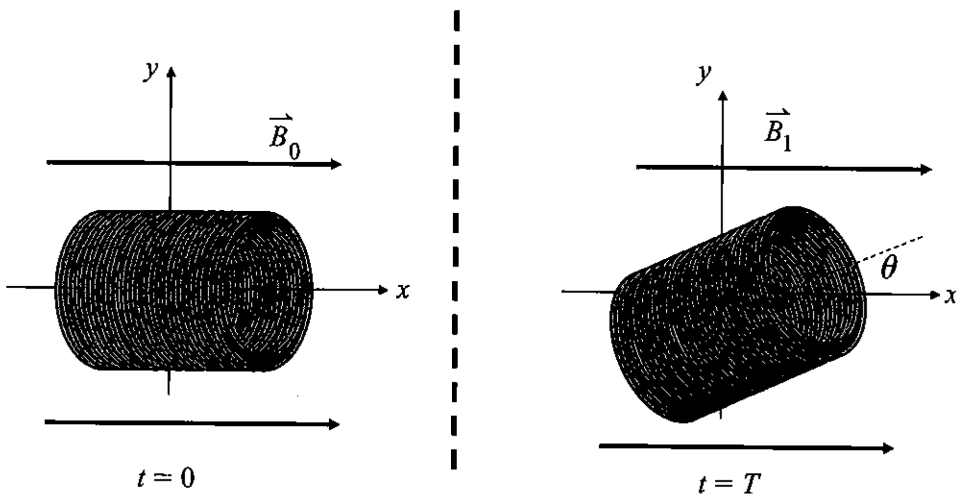

Figure 19

3. A length L of wire with cross sectional area a and resistivity ρ is wound around a hollow tube of radius R forming a coil. The two ends of the coil are connected and the coil is then aligned so that its axis lies along the x-axis of a coordinate system. A spatially uniform magnetic field B_0 parallel to the x-axis exists in this region. In a time T, the field strength changes to B_1 and the coil is rotated θ above the x-axis.

(a) In terms of L and R, how many loops are in the coil?
(b) What is the average induced voltage in the coil over the time T?
(c) What is the average current that flowed during the time T?
(d) The answer to (c) is independent of L, the length of the wire. Explain why this is so in physical terms.

Answers and Explanations

Multiple Choice

1. The answer is D. The area of the loop is 24×10^{-4} m^2. Since $\Phi = BA$, the result follows by multiplying by 0.5 T.

2. The answer is C. Moving the loop parallel to the field won't change how much of the field cuts through the loop. All other possibilities given will change the flux, either by changing the field in the loop (A and D), the area (B), or the orientation of the field with respect to the area (E). Changing the flux will induce a current.

3. The answer is E. The changing flux through either loop is $\frac{\Delta B}{\Delta t}(\pi a^2)$. Since $a < s$, whatever flux cuts through the smaller loop will also cut through the larger loop. While the two loops will have the same induced voltage, in general they will have different currents induced because in general they will have different resistances. Within each loop, different electric fields will be induced as well, but only C-level students need to address this level of complexity.

4. The answer is C. As the wire cuts across the field, a voltage of $V_{\text{in}} = BLv$ is induced between the ends of the wire. All of these quantities are constant.

5. The answer is B. The induced voltage from the changing flux will produce a current $I = \frac{V_{\text{in}}}{R} = \frac{\frac{\Delta B}{\Delta t}(LW)}{R}$. Solving for $\frac{\Delta B}{\Delta t}$ gives the result.

6. The answer is B. Since the field is *decreasing* into the loop, by Lenz's law the induced current will flow to try to *increase* the field into the loop. The loop right-hand rule then gives a clockwise direction for the current flow.

7. The answer is A. As the loop moves closer to the wire, the field is getting stronger. This field points into the paper at the position of the loop, so the flux is increasing into the paper. Lenz's law then tells you that the induced current will create a field that points out of the paper, and the loop right-hand rule gives a counterclockwise direction.

8. The answer is D. This is the force direction that will oppose the change. You can analyze the segments of the loop to see how this happens. From question 7 you know the current flows counterclockwise. The two shorter-width sections will experience equal but opposite ilB forces. The length section closer to the long wire will feel an ilB force directed away from the long wire, using the force right-hand rule. While the other length section feels a force toward the long wire, the field is weaker at its position, so the net force is away.

9. The answer is A. At terminal speed, the work done by gravity will not increase the KE of the rail so all the work done by gravity will have to appear as thermal energy in the resistor. The power supplied by gravity is $P = Fv = mgv_T$.

10. The answer is D. As the north pole approaches from the left, the field will point left to right through the loop, and it will be getting stronger. To oppose this change, the loop will have to create a field that points right to left, and this is achieved with a counterclockwise current. As the magnet exits the loop, the field direction is determined by the closer south pole, so the field still points left to right through the loop, but now it is getting weaker. To oppose this change, a the loop will have to create a field that points left to right, achieved by a clockwise current.

FREE RESPONSE

1. (a) The current will flow clockwise around the rectangular loop. The area of the loop is getting larger, causing the flux out of the loop to increase. By Lenz's law, the induced current will flow to oppose the increase in flux, and this can be accomplished by creating a field *into the page*. The right-hand rule then gives the clockwise direction.

   (b) The induced voltage is $V_{in} = BLv$. The induced current will then be $I = \frac{BLv}{R}$, since the voltage drop across the resistor is V_{in}.

   (c) The moving rail will feel a force to the left because the current I flowing in the rail is experiencing the magnetic field $\vec{B}$. This force is $F_{left} = ILB$. An equal force will have to be applied to the right to keep the rail moving at constant speed. Substituting for I, you have

$$F = \frac{B^2L^2v}{R}$$

   (d) At the instant the force is removed, the rail is moving at speed v and has kinetic energy $\frac{1}{2}mv^2$. Conservation of energy then says that all of this energy will eventually appear as thermal energy in the resistor, so the answer is $\frac{1}{2}mv^2$.

2. (a) At $t = 4$ s, the field is 0.3 T. The area of the loop is $(0.6\text{ m})^2 = 0.36\text{ m}^2$, so

$$\Phi_B = BA = 0.3(0.36) = 0.108 \text{ webers}$$

(b) At $t = 5$ s, the field is changing at the rate of $\frac{0.3}{2} = 0.15\frac{\text{T}}{\text{s}}$. This is the slope of the last line segment in the graph. Since the area doesn't change, the flux will be changing at the rate

$$\frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta B}{\Delta t} A = 0.15(0.36) = 0.054 \text{ V}$$

This is the induced voltage by Faraday's law.

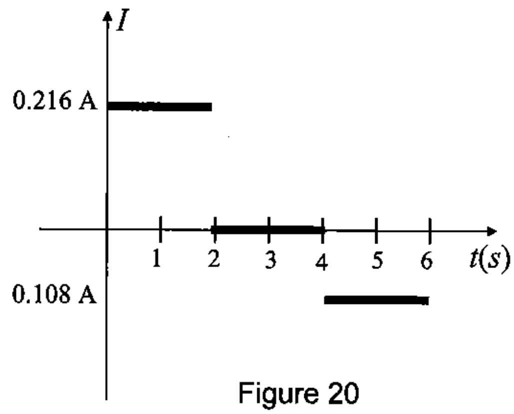

Figure 20

(c) Over the first 2 s, the field changes at the rate of $\frac{0.6}{2} = 0.3\frac{\text{T}}{\text{s}}$ (slope), and the induced voltage will be

$$\frac{\Delta \Phi}{\Delta t} = \frac{\Delta B}{\Delta t} A = 0.3(0.36) = 0.108 \text{ V}$$

The induced current then will be $I = \frac{V_{in}}{R} = \frac{0.108}{0.5} = 0.216$ A. By Lenz's law, this current will flow in such a way as to oppose the change in flux created by the decreasing field. This means the current will create a field in the + direction, and the right-hand rule then gives a counterclockwise direction for the flow. This is the + current direction.

Over the interval from 2–4 s, there's no change in flux, so the current will be 0. Over the last 2 seconds, you can see that the slope is just half of the slope over the first 2 seconds, so the induced current will be just half as much. In this case, however, the field is increasing, causing the flux up through the loop to increase. The induced current will flow to oppose this change in flux; this means that the current must flow clockwise, creating its own field down through the loop. This is the negative current direction.

3. (a) Since each loop in the coil has a circumference $2\pi R$, the number of loops is

$$N = \frac{L}{2\pi R}$$

(b) The initial flux is $NB_0 (\pi R^2)$ since the field is perpendicular to the plane of the loops. The part of the field that is perpendicular to this plane after time T is $B_1 \cos\theta$, so Faraday's law gives the average induced voltage.

$$V_{in} = \frac{\Delta\Phi}{\Delta t} = \frac{N\pi R^2(B_1\cos\theta - B_0)}{T} = \frac{LR(B_1 \cos\theta - B_0)}{2T}$$

(c) Since the resistance of the length of wire is $r = \rho\frac{L}{a}$, Ohm's law gives the average current over the time interval.

$$i = \frac{V_{in}}{r} = \frac{aR(B_1\cos\theta - B_0)}{2\rho T}.$$

(d) As L increases, more loops can be formed implying a greater induced voltage, but increasing L also increases the resistance by the same factor. The net effect is that the ratio of voltage to resistance stays the same.